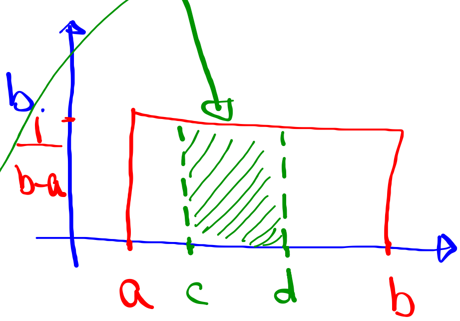
Statistics

Summer 2021

Lecture 13

Uniform Prob. dist. for all values from a to b.

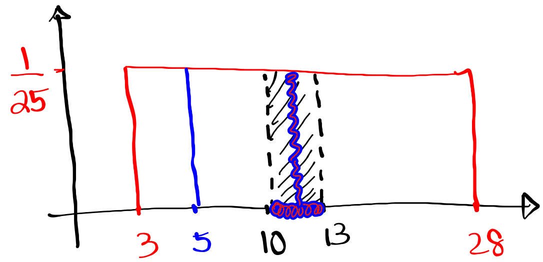
- Dist. is rectangular
- length goes from a to b.
- width is $\frac{1}{b-a}$
- $P(x=c)=0$
- $\boxed{P(c<x<d)}$ is the corresponding area within the rectangular graph.



Consider a uniform Prob. dist. for all Values from 3 to 28.

$P(x=5) = 0$

Line Zero Area



$P(10 < x < 13)$

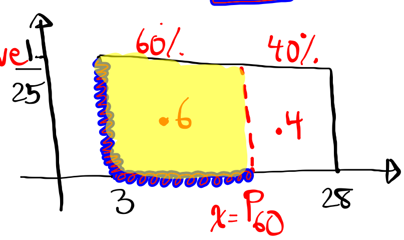
$= (13-10) \cdot \frac{1}{25}$

$= \boxed{\frac{3}{25}}$

Find $x = P_{60}$

60% below

40% above

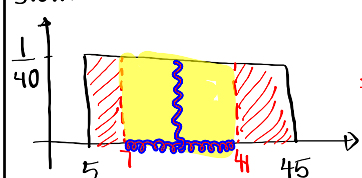


$(x-3) \cdot \frac{1}{25} = .6$

$x - 3 = 25(.6)$

$\boxed{x=18}$

Consider a uniform Prob. dist for all values from 5 to 45.



$P(x<7 \text{ or } x>41)$

$= 1 - P(7<x<41)$

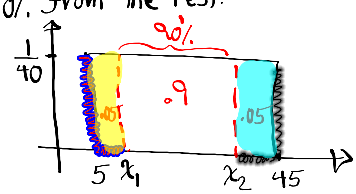
$= 1 - (41-7) \cdot \frac{1}{40}$

$= 1 - \frac{34}{40} = \frac{6}{40} = \boxed{\frac{3}{20}}$

Find two x-values that Separate the middle 90% from the rest.

$1 - .9 = .1$

$.1 \div 2 = .05$



$(x_1 - 5) \cdot \frac{1}{40} = .05$

$x_1 - 5 = 40(.05)$

$x_1 - 5 = 2$

$\boxed{x_1 = 7}$

$(45 - x_2) \cdot \frac{1}{40} = .05$

$45 - x_2 = 40(.05)$

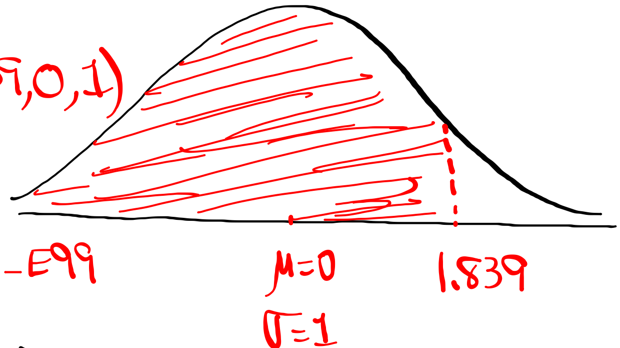
$45 - x_2 = 2$

$\boxed{x_2 = 43}$

$P(Z < 1.839)$

$= \text{normalcdf}(-E99, 1.839, 0, 1)$

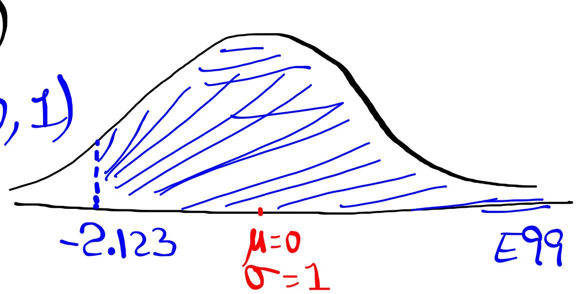
$= \boxed{.967}$



Find $P(Z > -2.123)$

$= \text{normalcdf}(-2.123, E99, 0, 1)$

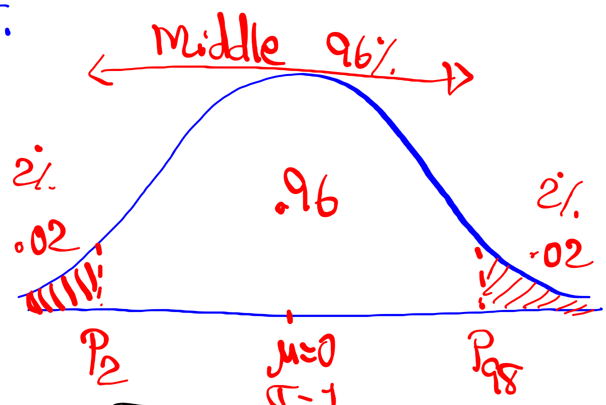
$= \boxed{.983}$



Find two Z-values that separate the middle 96% from the rest.

$1 - .96 = .04$

$.04 \div 2 = .02$



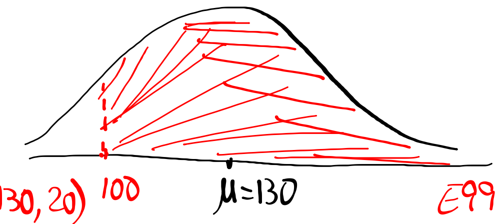
Left Area

$Z_1 = P_2 = \text{invNorm}(.02, 0, 1) = \boxed{-2.054}$

$Z_2 = P_{98} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

Consider a normal Prob. dist with $\mu = 130$ & $\sigma = 20$.

$P(x > 100)$

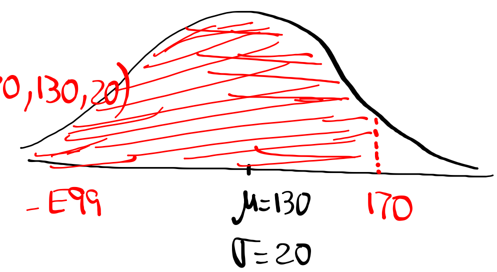


$= \text{normalcdf}(100, E99, 130, 20)$

$= \boxed{.933}$

$P(x < 170)$

$= \text{normalcdf}(-E99, 170, 130, 20)$



$= \boxed{.977}$

Find $x = Q_3$, Round to a whole #.

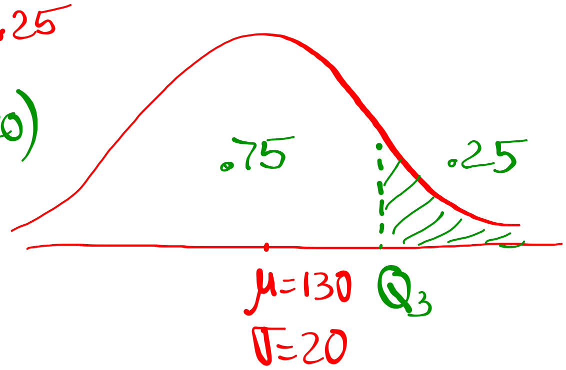
75% below — Left Area — .75

25% above — Right area — .25

$x = \text{invNorm}(.75, 130, 20)$

$= 143.490$

$\approx \boxed{143}$



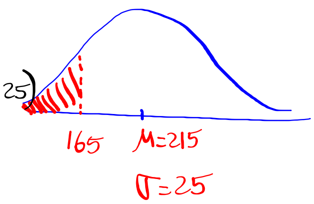
Total points scored in all NBA games are normally distributed with $\mu = 215$ & $\sigma = 25$

If we randomly select one game, find the Prob. that the total score is below 165 pts.

$P(x < 165)$

$= \text{normalcdf}(-E99, 165, 215, 25)$

$= \boxed{.023}$

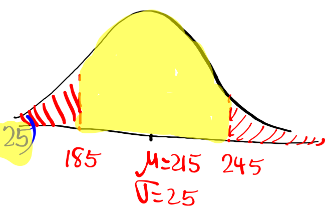


Find the Prob. that total pts is below 185 or above 245.

$P(x < 185 \text{ or } x > 245)$

$= 1 - \text{normalcdf}(185, 245, 215, 25)$

$= \boxed{.230}$



Central Limit Theorem:

$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

Consider $N(6275, 400)$ for randomly selected groups of 4, find

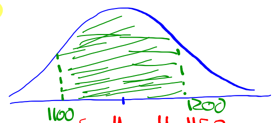
$\mu_{\bar{x}} = \mu = \boxed{6275}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{400}{\sqrt{4}} = \boxed{200}$

SAT Scores are normally distributed with $\mu = 1150$ and $\sigma = 75$.

If we randomly Select groups of 4 SAT exams, find the prob. that their mean is between 1100 & 1200.

$P(1100 < \bar{x} < 1200)$

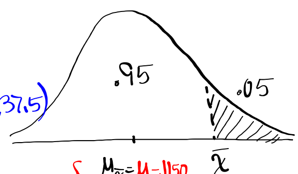


$= \text{normalcdf}(1100, 1200, 1150, 37.5)$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 1150 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{75}{\sqrt{4}} = 37.5 \end{cases}$

$= \boxed{.818}$

For randomly selected groups of 4 SAT exams, find $\bar{x}$ that separates the top 5% from the rest.



$\bar{x} = \text{invNorm}(.95, 1150, 37.5)$

$= 1211.682$

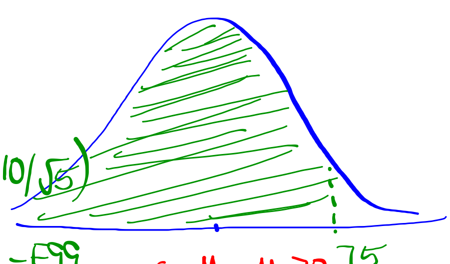
$\bar{x} \approx 1212$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 1150 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{75}{\sqrt{4}} = 37.5 \end{cases}$

Speed of Cars on freeway are normally distributed with mean of 70 mph and Standard dev. of 10 mph.

If we randomly Select 5 Cars, find the Prob. that their mean Speed is below 75 mph.

$P(\bar{x} < 75)$

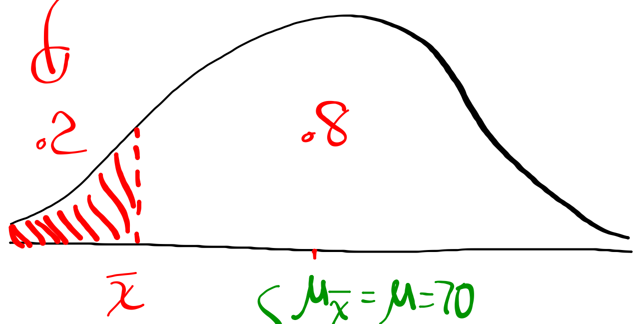


$= \text{normalcdf}(-E99, 75, 70, 10/\sqrt{5})$

$= \boxed{.868}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 70 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{5}} \end{cases}$

For randomly selected group of 5 cars, find their mean speed that separates the bottom 20% from the rest.



$\bar{x}$ = invNorm(.2, 70, 10/$\sqrt{5}$)

= 66.236 ≈ $\boxed{66}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 70 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{5}} \end{cases}$

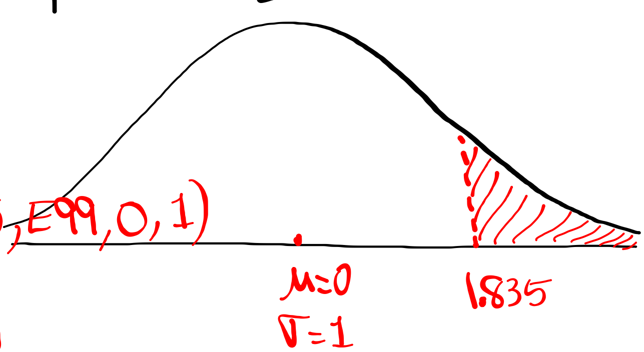
SG 22

Z Dist

- Bell-Shape
- Symmetric
- Total Area=1
- $\mu=0$, $\sigma=1$

Use normalcdf & invNorm

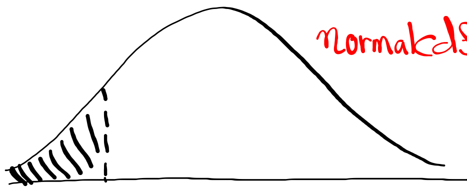
Find twice the area to the right of Z=1.835.



2*normalcdf(1.835, E99, 0, 1)

= $\boxed{.067}$

Find the shaded area below:



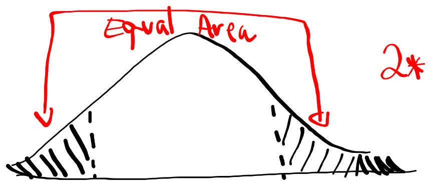
$Z = -2.345$

normalcdf(−E99, −2.345, 0, 1)

$= \boxed{.010}$

.0095

Find the shaded area below



$Z = -1.785$ $\quad$ $Z = 1.785$

2∗ normalcdf(1.785, E99, 0, 1)

$= \boxed{.074}$

1−normalcdf(−1.785, 1.785, 0, 1) $= \boxed{.074}$

SG 22

T Dist

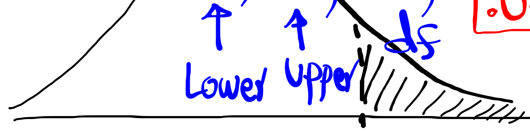
- Bell-shape
- Symmetric
- Total Area=1
- $\mu = 0$ & σ unknown
- It comes with degrees of freedom

Use tcdf & invT

Find the shaded area below with df = 9.

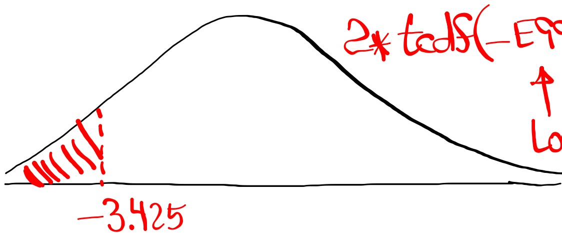
tcdf(2.035, E99, 9) $= \boxed{.036}$

Lower Upper df



$t = 2.035$

Find twice the area to the left of $t=-3.425$ with $df=15$.



$2 * \text{tcdf}(\underset{\text{Lower}}{-E99}, \underset{\text{Upper}}{-3.425}, \underset{df}{15})$

$= \boxed{.004}$

Find $P(t<-1.8 \text{ OR } t>1.95)$ with $df=19$.

$= 1 - \text{tcdf}(-1.8, 1.95, 19)$

$= \boxed{.077}$

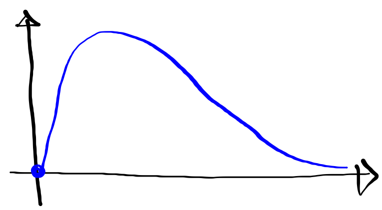
SG 22

Chi-Square dist.

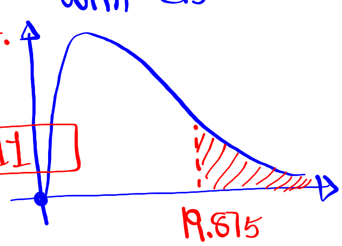
χ^2 dist

- dist. stats at 0,
- Skewed to the right
- Not Symmetric
- Total area = 1
- It also comes with df.

Use $\chi^2\text{cdf}$

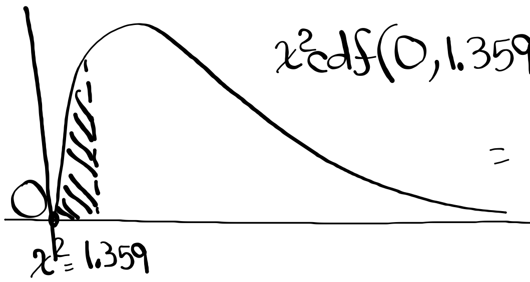


Find $P(\chi^2 > 19.875)$ with $df = 8$.



$\chi^2\text{cdf}(\underset{\text{Lower}}{19.875}, \underset{\text{Upper}}{E99}, \underset{df}{8}) = \boxed{.011}$

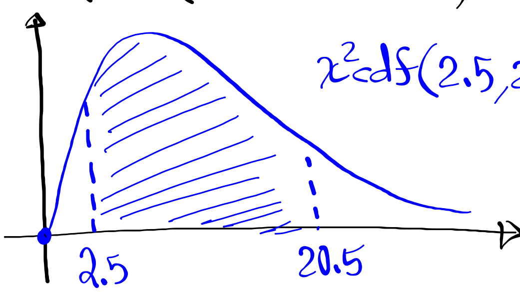
Find the shaded area below with $df = 10$



$\chi^2cdf(0, 1.359, 10)$

$= \boxed{6.9 \times 10^{-4}}$

Find $P(2.5 < \chi^2 < 20.5)$ with $df = 12$.



$\chi^2cdf(2.5, 20.5, 12)$

$= \boxed{.940}$

F- Dist

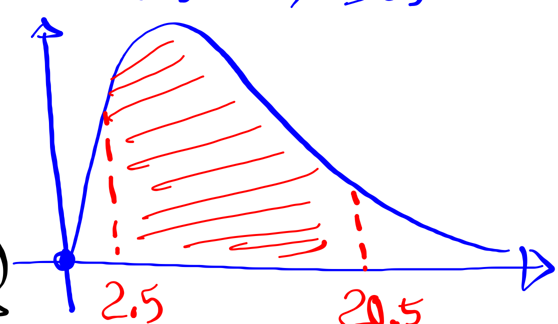
SG 22

- It is Similar to χ^2- Dist.
- Numerator df Ndf
- Denominator df Ddf

Use Fcdf

$P(2.5 < F < 20.5)$

$Ndf = 5$, $Ddf = 25$



Fcdf(2.5, 20.5, 5, 25)

Lower Upper Ndf Ddf

$= \boxed{.057}$

Find the shaded area below with

Ndf = 3 & Ddf = 23

$$\text{fcdf}(\underset{\text{Lower}}{0}, \underset{\text{upper}}{.987}, \underset{\text{Ndf}}{3}, \underset{\text{Ddf}}{23}) = \boxed{.584}$$